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Stability of locomotives and cars in motion

by academician V.A. LAZARYAN,
Ukrainian Academy of Sciences, Kiev

SUMMARY

High-speed running of trains requires complex studies of problems of ensuring stability of rolling stock and track, as well as, safety and comfort.

Theoretical studies are becoming vital for fundamental solutions of problems of stability of undisturbed motion of rail vehicles. The paper describes some theoretical methods for the solution of these dynamic problems confirmed by tests with a high-speed mobile laboratory.

To ensure good running characteristics of rail vehicles, it is necessary that their motion be stable in a given range of speeds.

The instability of undisturbed motion of rail vehicles results in the appearance of horizontal transverse movements, limited by the flange thrust against the rail. At comparatively low speeds, the forces originating from flange pressure against rail do not attain dangerous magnitudes for track of modern design. However, such transverse movements deteriorate the running smoothness, increase stresses in rolling stock and track and this leads to increased wear of running parts and rails. With increase in speed of unstable vehicles, the horizontal forces acting against the track build up at such a rate that derailment is imminent and the track is destroyed. Therefore, it is important to design the locomotives and cars so that their undisturbed motion is, for a given speed range, asymtotically stable in accordance with Lyapunov's theory. Lately intensive research is taking place of structures which ensure stable movement of rail vehicles and significantly prolong service life of running gear, particularly the wheelsets [1].

The theoretical studies of stability of rail vehicles, moving over track, are vital for the solution of this problem and are based on the general theory of stable motion created by Lyapunov at the end of 19th century and further developed by Soviet scientists. The fundamentals of the theory of stability of undisturbed motion of rail vehicles are given in [2, 3].

The stiffness of spring suspension elastic elements of modern locomotives and cars is much lower than of other structural elements. Therefore, they may be considered as a system of absolutely rigid bodies, connected by deformed and rigid elements. In some cases, the final stiffness not only of the spring suspension but of other structural elements should be taken into account and consider these elements as deformed bodies.

Locomotive and car wheels have tapered rolling surfaces and are rigidly mounted on axles forming the wheelset. If such wheelsets are joined by the frame into bogies then pure wheel rolling without sliding is impossible. Therefore, constraints imposed on the system would be holonomic. The number of independent generalized coordinates in such a case equals the number of degrees of freedom of the mechanical system under consideration. The number of degrees of freedom depends on the design and the idealization used. When constructing the mathematical model, it is always necessary to simplify to a certain degree the properties of the actual system. The system simplified by idealization is called the calculation scheme and it should be choosen so as to, as completely as possible, reflect the properties of the real system. Oversimplification of the calculation scheme could give quite wrong results. Therefore, rail vehicles should be considered as nonlinear mechanical systems with a large number of degrees of freedom.

The differential equations of motion of the mechanical system are given as second order Lagrange equations :

$$\frac{d}{dt} \cdot \frac{\delta T}{\delta \dot{q}_i} - \frac{\delta T}{\delta q_i} = Q_i ,$$

where :

q_i and $\dot{q}_i$ = generalized coordinates and generalized speed;

T = kinetic energy of system;

Q_i = generalized force corresponding to the generalized coordinate q_i.

The kinetic energy is defined as the sum of kinetic energies of bodies, forming the system, and the kinetic energy of each body is defined by the König theorem.

The generalized coordinates which define the configurations of the considered mechanical systems are both position and cycle coordinates. The generalized coordinate q_i is a cycle coordinate if the corresponding generalized force Q_i equals zero, and all other generalized forces and the kinetic energy of the system are independent of this coordinate. All other generalized coordinates are the position coordinates.

Let us assume that the holonomic constraints imposed on the system and its configuration are defined by n generalized coordinates $q_1, q_2, \dots, q_n$ and r of them $(r < n)$ are position (noncycle), and the other $n - r$ are cycle. Let us designate cycle coordinates by $q_{r+1}, q_{r+2}, \dots, q_n$. In such a case, the generalized forces $Q_{r+1}, Q_{r+2}, \dots, Q_n$ are equal to zero and T does not depend on generalized coordinates $q_{r+1}, q_{r+2}, \dots, q_n$, therefore, for $i > r$:

$$\frac{\delta T}{\delta q_i} = 0\,, \quad \frac{d}{dt}\cdot\frac{\delta T}{\delta \dot{q}_i} = \frac{d}{dt} p_i = 0\,,$$

where :

p_i = generalized impulse.

Therefore,

$$\frac{\delta T}{\delta \dot{q}_i} = p_i = \alpha_i = \text{const} \quad (i = r+1, r+2, \dots, n)\,,$$

i.e. cycle impulses have constant values. These are the first integrals of the Lagrange equations, called « cycle integrals ».

They give a system of $n - r$ equations by which all cycle speeds may be expressed through the position coordinates and speeds, and also constants α_i. For the determination of position coordinates, we obtain r differential equations.

An effective method of eliminating cycle coordinates is the Rauth method [2]. If the Rauth function :

$$R = T - \sum_{i=r+1}^{n} \frac{\delta T}{\delta \dot{q}_i}\dot{q}_i = T - \sum_{i=r+1}^{n} \alpha_i \dot{q}_i\,,$$

be introduced into the Lagrange equation, the Lagrange equation, for the position coordinates, will assume the form :

$$\frac{d}{dt}\cdot\frac{\delta R}{\delta \dot{q}_i} - \frac{\delta R}{\delta q_i} = Q_i \quad (i = 1, 2, \dots, r),$$

and cycle coordinates will be determined by quadraturing :

$$q_i = -\int \frac{\delta R}{\delta \alpha_i} dt \quad (i = r+1, r+2, \dots, n).$$

It may be shown that, even with stable constraints, when the kinetic energy of the system is a uniform second order function of generalized speeds, the Rauth function will be :

$$R = R_2 + R_1 - R_0\,,$$

where :

R_2 = the member of the second,

R_1 = of the first, and

R_0 = the zero order referred to the position speeds.

Therefore, the differential equations of movement will additionally include generalized centrifugal forces and gyroscopic forces of the $\Gamma_{ij}\,\dot{q}_j$ form and besides, an apparent change in mass of the system will take place, since the coefficients in the expression R_2 will differ from these in expression T [2, 3].

The generalized coordinate, defining the movement of the locomotive or car along track axis is a cycle coordinate. After eliminating this coordinate by the Rauth method, we obtain the equation of disturbed movement of this coordinate in the Rauth form. When the undisturbed motion is rectilinear, the influence of the cycle coordinate appears only in the apparent change of mass of system. The equations of disturbed motion in this case may be obtained as a second order Lagrange equation by eliminating directly from the differential equations of motion the second derivative of the cycle coordinate [2, 3].

The Lagrange equations of the second order in this case may be conveniently written as :

$$\frac{d}{dt}\cdot\frac{\delta T}{\delta \dot{q}_i} - \frac{\delta T}{\delta q_i} + \frac{\delta \Phi}{\delta \dot{q}_i} + \frac{\delta \Pi}{\delta q_i} = Q_i \quad (i = 1, 2, \dots, n)$$

where :

Π = potential energy of system;

Φ = the function of energy dissipation;

Q_i = generalized force corresponding to generalized coordinate q_i, which has no potential and is not determined by means of the function of energy dissipation.

Forces Q_i are defined as coefficients of the variations of corresponding generalized coordinates in an expression for possible work of the applied forces.

The forces of interaction of wheels and rails in their contact planes are forces external to the moving system. These forces are usually determined by means of the theory of pseudo-rolling proposed by Carter. According to this theory, the tangential forces acting on the wheels are proportional to the relative slipping of contact surfaces of wheels and rails. Components X_i and Y_i of the forces, acting on wheel with a number i, parallel and normal to axis of track are determined by formulae :

$$X_i = -F\,\varepsilon_{xi}\,, \quad Y_i = -F\,\varepsilon_{yi}\,,$$

where :

ε_{xi} and ε_{yi} = dimensionless characteristics of slipping along and across the track, and

F = pseudoslipping coefficient proportional to square root of wheel radius and its pressure on the rail.

If dry friction is taken into account at the slipping section of the contact area, the component forces acting on the wheels will be :

$$X_i = -f\,\varepsilon_{xi} - f_1\,\varepsilon_{xi}^2\,,$$

$$Y_i = -f\,\varepsilon_{yi} - f_1\,\psi\,\varepsilon_{xi}^2\,,$$

where :

ψ = the small angle of wheelset turning relative to the vertical axle, and

f and f_i = constants, which depend on the friction coefficient, elastic constants of material and pressure on rail [2, 3].

When the stability of steady motion is to be studied, the system of differential equations of disturbed motion will be autonomous. In the normal Cauchy form, this system has the form :

$$\frac{dx_s}{dt} = X_s\,(x_1, x_2, ..., x_n) \quad (s = 1, 2\ ..., n)\,,$$

where :

x_s = the vector components $x = [x_1\ x_2\ ...\ x_s\ ...\ x_n]'$ of phase coordinates.

The stability of undisturbed motion of rail vehicles is at present being studied mainly by using the first (linear) approach. The direct (the second) Lyapunov method for solving this problem, has not yet been used, because the construction of Lyapunov's function for railway vehicles, which are complex systems with a great number of degrees of freedom, is very difficult.

Lyapunov himself considered as his main achievement his theorems of stability and unstability for the first approximation. A.M. Lyapunov showed that, in some cases, the substitution of nonlinear equations of disturbed motion by linear equations leads to an entirely different problem. The theorems of stability and instability for the first approximation thus clearly defined the range of problems in which the first approximation gives the solution to the initial nonlinear problem. In all cases, when the right-hand parts of equations of disturbed motion could be expanded into power series, it is possible, in a sufficiently small region D of the coordinate origin to linearize this system of equations eliminating after expanding all terms of the second and higher orders.

The system of equations of disturbed motion in such case will be :

$$\dot{x} = A\,x\,,$$

where :

A = square matrix, the elements of which depend on the structure and parameters of the considered mechanical system.

The study of stability of movement is reduced to the solution of the problem of linear algebra of determining the eigen matrix value A. When studying the stability of motion of rail vehicles, the matrix A is non-symmetrical and has not only real but also complex proper numbers. If among them there are no numbers with a zero real part, then, for sufficiently small initial disturbances and current deviations, the problem of stability or unstability of undisturbed motion of the initial nonlinear system is completely solved by the first approximation. The cases when the proper numbers have zero real parts are special cases and require additional investigations.

The very important case here is that with linearization of equations of disturbed motion by expanding their right-hand parts into an exponential series and neglecting the nonlinear terms, the Lyapunov theorems of stability and unstability for the first approximation, are valid only with sufficiently small initial and current disturbances. However, unfortunately, there are no effective criteria for defining what are the values of sufficiently small disturbances. The Barbashin criteria though of considerable theoretical value, is of little use for practical calculations. The stability of undisturbed motion for any initial disturbances is called « stability as a whole ». The problem of « stability as a whole » is completely solved with the help of direct (the second) Lyapunov method based on the Barbashin-Krasovsky theorem.

To expand the limits of use of the first approximation and to use the method not only for small but also sufficiently large disturbances, it is proposed to replace the right-hand parts of the equations of disturbed motion by the best linear approximations according to Chebyshev in the given area D of the origin of coordinates. With this linearization in the D area of the origin of coordinates, the first approximation stability theorems of Lyapunov will become valid.

A number of checks by numerical integrations of initial equations of disturbed motion confirm this assumption; however, it is necessary to find its rigid substantiation. The use of Chebyshev's linear approximation permits to reduce the critical cases to noncritical [3, 4]. The most effective method of realization of the Chebyshev linear approximation by electronic computers is the Remes second algorithm.

To solve the complete problem of eigen values, i.e. to define all matrix A eigen values of linearized system coefficients of differential equations of disturbed motion, the QR algorithm is used [5].

With a linear system of differential equations it is more convenient to make point and contact transformations of coordinates in which the system of equations are broken up into separate independent subsystems. When rail vehicle motion stability is studied, such differentiation is very useful since the order of the equation systems of disturbed motion is sufficiently high : of the 20th - 30th in simple and 50th and higher orders in more complicated cases. The possibility of differentiation of the system of equations to subsystems usually corresponds to some obvious or hidden symmetry of the mechanical system. In some cases, it becomes possible to choose the generalized coordinates right away, and get independent subsystems of equations [2, 3]. In the general case, the problem is to reduce the system of linear differential equations to a maximum possible number of independent subsystems by multiplying its left-hand part by a nonsingular matrix and substitute the variables $x = Sy$, where S is also a nonsingular matrix.

The use of analogue computers has proved successful for solving problems of oscillations and stability of mechanical system motion. Their solution with a system of linear differential equations of form $\dot{x} = A\,x$ is simple. If undisturbed motion of the mechanical system is asymptotically stable, all eigen value matrix numbers A of the linearized system of equations of disturbed motion have negative real parts. In this case, the solutions of computer equations with correct choice of scale coefficients do not exceed the limits of change of computer variables in the model. If the undisturbed motion is unstable, then the real part of at least one of the proper numbers of matrix A is positive. In this case, the number of solutions increase without limit and exceed the limits of change of computer variables in the model and the solution becomes impossible. To obtain a solution with a model also for such a case, we substitute the variables $x = x^* e^{\alpha t}$, where x and x^* are vectors of phase coordinates, and α is a real number. Since $\dot{x}^* = e^{\alpha t}\dot{x} + \alpha\, x\, e^{\alpha t}$, after substitution of variables the system of differential equations $\dot{x} = A\,x$ will assume the form

$$\dot{x}^* = (A - \alpha\, E)\, x_s^*\,,$$

where :

E = the unit matrix.

The eigen values of numbers of matrix $A - \alpha\, E$ will be :

$$\lambda_i^* = \lambda_i - \alpha\,,$$

i.e. the new eigen values of numbers (roots of the characteristic equation) are shifted in respect to their previous values λ_i to the left by the amount α. This method is called the « shift of roots ». Let us assume that the biggest real part of eigen numbers λ_i will be h_{max}. Having chosen $\alpha > h_{max}$, we get all λ_i^* with negative real parts. The system of differential equations $\dot{x}^* = (A - \alpha\, E)\, x^*$ will become asymtotically stable and it may be easily solved by an analogue digital computer. Value may be found by solving the system $\dot{x} = A\,x$ by a computer [6].

The method of shifting of roots is used for determining the structure and parameter values of the physical systems whose motion is stable. Let the linearized system of equations of disturbed motion obtained either in the Rauth or Lagrange form be :

$$M\ddot{q} + B\dot{q} + Cq = 0\,,$$

where :

M = diagonal matrix of inertial coefficients;

B and C = the matrix coefficients at generalized speeds and coordinates; and

q, $\dot{q}$ and $\ddot{q}$ be the h-dimensional vectors of generalized coordinates, speeds and accelerations.

Matrix elements M, B and C are parameter functions $\alpha_1, \alpha_2, \ldots, \alpha_n$ systems and do not depend apparently on time. Let us consider the phase coordinates. Let x and $\dot{x}$ be vectors of phase coordinates and speeds. A normal Cauchy form of equations of disturbed motion is $\dot{x} = A\,x$, where A may be represented in a form of a block matrix :

$$A = \begin{bmatrix} 0 & E \\ -M^{-1}C & -M^{-1}B \end{bmatrix}.$$

Let the zero solution of the system of differential equations $\dot{x} = Ax$ be unstable, then, among the eigen-value numbers of matrix A there are numbers with a positive real part. Let us construct a mechanical system, which is structurally close to the initial and select such parameter values of this system for which its motion will be stable in a given range of speeds. With this aim in view, we transform the coordinates $x = e^{\alpha t}\, y$, where α = real number, the parameter of the shift of roots. The system of equations of disturbed motion will become

$$\dot{y} = (A - \alpha\, E)\, y\,,$$

where :

$$A - \alpha\, E = \begin{bmatrix} -\alpha\, E & E \\ -M^{-1}C & -(M^{-1}B + \alpha\, E) \end{bmatrix}.$$

Returning from normal Cauchy form to equations of the second order, we find

$$M\ddot{q} + (B + 2\alpha M)\dot{q} + (C + \alpha B + \alpha^2 M)q = 0\,.$$

This is the mathematical model of the mechanical system, the motion of which is stable if α is chosen in such a way that among the eigen matrix number $A - \alpha E$ there are no numbers with a positive real part. Elements corresponding to addends $2 \alpha M$ and $\alpha B + \alpha^2 M^2$ should be inserted into the structure of the new system, and the parameters changed correspondingly. Since there are no general methods of synthesis of physical systems by their mathematical models, to add new elements to the structure of the system and change its parameters, the method of successive approximations should be used. It should be emphasized that the construction of a stable system often requires not only change in parameters but also of the structure.

Then there is the problem of determining such values of parameters of the mechanical system for which the degree of stability would be maximum. According to the Lyapunov theorem concerning stability to the first approximation, it is necessary that all the eigen numbers of matrix A have negative real parts, i.e. that the condition

$$h_{max} = \max \ Re \ \lambda_i < 0$$

be fulfilled. The value of h_{max}, in this case, defines the reserve of stability of the system. This value continuously depends on the parameters $\alpha_1, \alpha_2, ..., \alpha_m$ of the system, which is considered as independent variables and the value h_{max} as a function of these variables [3]. Consequently, the next problem for nonlinear programming can be formulated as follows:

$$h_{max}(\alpha) \rightarrow \min_{\alpha \in D}, \qquad D = \{\alpha \in E_m : \alpha_j^- \leqslant \alpha_j \leqslant \alpha_j^+\},$$

where :

$\alpha = [\alpha_1 \alpha_2 ... \alpha_m]'$ = vector of optimizing parameters of the system;

D = region of their allowable values; and

α_j^- and α_j^+ = upper and lower limits of parameters, defined by structural, technological and other considerations.

For solving this problem, a series of algorithms has [7, 8, 9,] been elaborated, both regular and those which use the idea of random probing. The target function in the given range of parameters may be multi-extreme, therefore the local probing algorithm was used in combination with choice of initial conditions. Rational parameters of spring suspension of a series of passenger cars and electric locomotives have been found with the help of the developed algorithms.

Some system parameters, for example, the coefficient of pseudo-sliding, wheel tread taper may change during operation, therefore it is of interest to develop such structures for which function $h_{max}(\alpha)$ would be least sensitive to the change of parameters. This problem may be solved by numerical methods, by minimising the penalty function of a form :

$$F(\alpha, K) = K(h_{max} - \xi)^2 + S(\alpha),$$

where :

K = penalty coefficient;

$\xi < 0$ = given reserve of stability

$$S(\alpha) = \max_{1 \leqslant i \leqslant s} \left[\sum_{j=1}^{l} \left(\gamma_j \frac{\delta \, Re \, \lambda_i}{\delta \alpha_i} \right)^2 \right]^{1/2}.$$

Here :

γ_j = weight coefficient;

S = number of eigen numbers, which satisfy the condition

$$Re \ \lambda_i \geqslant h_{max} - \varepsilon, \quad \varepsilon > 0.$$

Because of oscillations of rail vehicles the pressure on the wheelset axle changes with time, therefore the pseudo-slipping forces change too. If we take this into account, then the equations of disturbed motion will have alternating coefficients, i.e. the systems considered will turn out to be non-autonomous. The investigations show that even if the amplitudes of the change in wheelset axle pressure equal 0.7 of the nominal pressure for freight, passenger and metro cars, and electric trains and also for the high-speed laboratory test car (see below), the critical speed of the non-autonomous systems is not more than 10%. For this reason, only autonomous systems are considered.

Together with the study of vehicle motion stability by the first approximation, some constructive methods of lowering the orders of autonomous and nonautonomous systems of nonlinear differential equations [10] are used. These methods are based on Lyapunov reducing principles. The lowering of the order of the systems of equations is carried out by elimination of fast-damping solutions. If among the solutions there are fast-damping ones, then in the matrix spectrum of a linearized system of equations (together with eigen numbers, located near the imaginary axis, i.e. with small absolute values of real parts) there are proper numbers in the left semiplane far from the imaginary axis. These eigen numbers have larger absolute values of negative real parts.

To eliminate the fast-damping solutions, the initial system of differential equations of disturbed motion is linearized by means of the Chebyshev linear approximation and reduced to main phase coordinates. The main coordinates corresponding to fast-damping solutions are assumed to be equal to zero. This gives the possibility to express a part of the generalized coordinates through the others. The obtained expressions are substituted into the initial nonlinear system. This will essentially reduce the order of the system of equations describing the steady regime of oscillations. When the system is not asymtotically stable but has a stable limiting cycle, the method permits to determine the amplitude algebraically corresponding to the limiting cycle. Since the Chebyshev approximation is used, the elements of the matrix coefficients depend on the intervals of variation of phase-coordinates at which linearization is carried out. Varying the linearization ranges it is possible to find such amplitude values for which the maximum real part of the matrix eigen numbers in the linearized equations becomes equal to zero. It is these limits that define the stable limiting cycle. This method is very effective. Thus, for example from an autonomous system of nonlinear differential equations of the 32nd order with the help of which the stability of a car with a double spring suspension was determined, a 4th-order block containing nondamping solutions was separated.

The comparison of results of numerical integration, obtained for comparable initial conditions, has shown that the deviations in the solution of the 4th-order system from the solution of the initial system is 3% for frequency and 5% for limiting cycle amplitudes. When determining amplitudes of the limiting cycle algebraically the error does not exceed 7%. If instead of the best from the point of view of the Chebyshev linearization we were to use series expansion the errors increase five to seven times for the problems considered. This is because the Chebyshev linearization gives a more uniform approximation for the nonlinear functions in the entire region under consideration.

The applicability of this method to non-autonomous systems was checked by solving the problem of oscillations of a freight flat for high-speed rail traffic [11]. The motion stability of this vehicle was determined from the system of equations of the 32nd order. Running along a track was considered whose axis in the plan has sinusoidal deviations from a straight line :

$$y = d \sin \omega t, \quad \omega = 2 \pi v L^{-1},$$

where :

v = speed;

L = wave length.

The most unfavourable resonance case was considered and the order of the non-autonomous system was reduced from the 32nd to the second order. The deviation when integrating a complete and shortened system is 5.7%.

The developed theory for stability of non-disturbed motion of rail vehicles was checked experimentally on one of the railway sections. For this purpose, a high-speed laboratory car was used with jet traction [12]. It was designed so that its critical speed equalled 100 m/sec (360 km/h).

The conditions on the section and tractive effort of the jet engines allowed to achieve a speed up to 250 km/h. Within the entire range of speeds, from 0 to 250 km/h, the motion of the laboratory car was asymtotically stable. Only constrained transverse horizontal vibrations originate due to horizontal track iregularities. These vibrations were damped sufficiently quickly. Side pressures of wheels on the rails did not exceed 1.8 t. For comparison sake it should be mentioned, that when a loaded gondola on a standard bogie moves at a speed of 100 km/h a loss of stability results in side pressures on rails reaching 8 to 10 t.

To check the methods for determining the critical speed, for the first time in world practice, a special investigation was carried out. The body suspension design was changed so that the theoretically found critical speed was within the range of realized speeds. For this purpose, the support of the body was transfered from the side bearings to a properly lubricated central bearing, the dampers of the axle-box suspension were removed and the tread surface of all wheels were turned with a new profile with a taper of 1:10 instead of 1:20. With such changes in design, the critical speed should have been of about 140 - 150 km/h while the measured experimental values of critical speed turned out to be 155 - 170 km/h. Thus, the developed theory is in proper agreement with the specially carried out experiments.

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